International Trade and Macro: Solving PE sunk-cost models

Solving discrete models

- Goal: Solve and estimate a sunk-cost model
  1. **1.** Solve model
  2. **2.** Estimate model
- Focus on solution for now

Algorithm

1. Initial set up
   - Set parameter values
   - Construct grids; Discretize continuous stochastic processes
   - Initialize policy and value functions
2. Solve decision problem
   - Value/policy function iteration to convergence
   - Key output: Policy functions
3. Create simulated panel of data
   - Set initial firm states; Draw sequences of shocks
   - Use policy functions to model firm behavior, record panel
   - Use panel to compute moments in simulated data
4. Compare model-moments to data-moments
   - If moments match, finished
   - If moments do not match, update parameters, return to step 2.

1. Initial setup

- Parameters
  - θ = elasticity of substitution in demand
  - τ = tariff (constant for now, could be stochastic)
  - β = discount factor
  - δ = survival probability
  - f_0, f_1 = export entry, continuation costs
  - A process for z ($\bar{z}$, ρ, σ_ϵ)

$$\log(z') = (1-\rho)\log(\bar{z}) + \rho\log(z) + \epsilon$$

$$\epsilon \sim \text{iid } \text{N}(0, \sigma_\epsilon)$$

  - $\xi_H > \xi_L$ export variable costs (constant for now, could be stochastic)
  - $g = g(z)$ is the probability mass function of new producers

1. Initial setup

- Construct a grid for z
  - Equally spaced points
  - Importance-weighted: Use CDF of ergodic distribution
- Use Tauchen-like method to convert AR(1) to discrete Markov chain
- Precompute and store $\pi(x, z, \xi)$
- Initialize value and policy functions
  - $V^1(x, z, \xi)$ value function for exporter ($N_z \times N_\xi \times 2$)
  - $V^0(x, z, \xi)$ value function for non-exporter
  - $V(x, z, \xi)$ value of the firm (need two of these, old and new)
  - $X(x, z, \xi)$ export decision
  - Initialize V to something like $\pi(z, \xi)/(1 - \beta\delta\rho)$
- Ancillary functions: $l(x, z, \xi), ex(z, \xi)$. Compute after convergence.

2. Solve decision problem

- Value function iteration. Loop over z_i

$$V^1(x, z_i, \xi) = \pi(x, z, \xi) - xf_1 - (1-x)f_0 + \beta \sum_{z_j} V_{\text{old}}(1, z_j, \xi)\text{prob}(z_j|z_i)$$

$$V^0(x, z_i, \xi) = \pi(x, z, \xi) + \beta \sum_{z_j} V_{\text{old}}(0, z_j, \xi)\text{prob}(z_j|z_i)$$

$$V_{\text{new}}(x, z_i, \xi) = \max\left\{V^1(x, z_i, \xi), V^0(x, z_i, \xi)\right\}$$

- Check: $\|V_{\text{new}}(x, z_i, \xi) - V_{\text{old}}(x, z_i, \xi)\|$
- If not converged, set $V_{\text{old}}(x, z_i, \xi) = V_{\text{new}}(x, z_i, \xi)$, repeat
- Once converged, compute $X(x, z_i, \xi)$, $I(x, z_i, \xi)$, $ex(z_i, \xi)$

2. Decision rules: interpolation

- With a discrete choice, there is a cutoff for entry z_0, z_1 but this cutoff is generally between nodes.
- Thus small changes in parameters can lead to discrete changes in the mass of firms making the choice.
- This can lead to some instability in convergence or parameter estimation, especially with sparse grid.
- Solution: interpolate and randomize.
  - Find the cutoffs using the value functions.
  - Assume firms are distributed uniformly between the nodes and then let the decision rule be based on the share of firms that meet the threshold.

3. Simulate a panel

- We have the decision rules…
- Want to create a panel data set of firms in the stationary distribution
  1. $t = 0$: Create N_f firms, assign each a ξ and a z_0; all nonexporters
  2. $t = 1, \ldots, t = T$; $f = 0, \ldots, N_f$
     - Draw a z_t for firm f (use ergodic dist and uniform random)
     - Compute export decision, production, exports, etc.
  3. To avoid initial conditions problem, throw out first several hundred observations. Check that moments do not change (much) over the panel.
- Now we have a panel of data…
- If we structured our panel correctly we can **literally** use the same code we used on the data on the model panel.

3. Distribution dynamics

- Let $\mu_t(x, z_i, \xi)$ denote the mass of each type of firms.
- Vector μ_t evolves according to difference equation

$$\mu_{t+1} = \Psi_t \mu_t + m_t\, g_t, \qquad t = 0, 1, ...$$

  where $n \times n$ coefficient matrix Ψ_t has elements governing the exogenous and endogenous transitions

- Mass of firms at node (x, z_i, ξ) at $t + 1$ depends on transition probabilities and export entry and exit decisions of incumbents at t plus flow of new entrants

Speeding up the last step

- With linear law of motion for μ, the stationary distribution is linearly homogeneous in m
- In terms of the discretized system above

$$\boldsymbol{\mu} = \boldsymbol{\Psi}\boldsymbol{\mu} + m\mathbf{g} \qquad \Rightarrow \qquad \boldsymbol{\mu} = m(\mathbf{I} - \boldsymbol{\Psi})^{-1}\mathbf{g}$$

  where $\mathbf{I}$ is an identity matrix

- Two implications
  - no need to use simulations to find stationary distribution $\boldsymbol{\mu}$, just set up coefficient matrix $\boldsymbol{\Psi}$ (implied by $x^*(x, z_i, \xi)$) and calculate directly
  - only invert $(\mathbf{I} - \boldsymbol{\Psi})$ once, then just rescale by m
- We wrote this down as one $\mu_t(x, z_i, \xi)$ but obviously could write this down as several distributions $\mu_t^j(z_i, \xi)$ which allows for easier inversion.

Computing moments: two approaches

1. Simulate panel.
   - Sampling errors can be large, may require long burn ins.
   - Can be very slow.
   - But data is from finite samples.
2. Using decisions rules and ergodic distributions
   - Yields exact moments using integration and and some iterations of transition matrices.
   - Can be very fast.
   - No errors

Aggregate shocks

- No aggregate uncertainty here
- Make E_t an AR(1) process that affects all firms identically
- Need to discretize and add to the firm's state variables

$$V(x, z, \xi, E) = \pi(z, \xi, E) + \cdots + \sum_{z', E'}$$

- Easy to do in partial equilibrium; will typically overstate the effect of a foreign demand shock — price dynamics will attenuate also misses out on rich interactions between other macro variables (income, wages, interest rates, ...)

Endogenous innovation models

- Firm efficiency is exogenous
- Recent work emphasizes endogenous efficiency. Introduce investment, I_i, that affects distribution of idiosyncratic productivity

$$F\left(a_j | a_i, I_i\right)$$

- Model the cost as increasing in current productivity, $\chi a_i^{\theta} I_i$ with $\theta > 1$
- Luttmer (07), Klette & Kortum (04), Atkeson & Burstein (09), Lenz & Mortenson (07)

Solution methods

- Moll: https://benjaminmoll.com/codes/
- Winberry: https://www.thomaswinberry.com/research/winberryAlgorithm.pdf
- Terry: https://onlinelibrary.wiley.com/doi/abs/10.1111/jmcb.12414
- Mongey: http://www.simonmongey.com/teaching--notes.html
- Kirkby: https://www.vfitoolkit.com/